ADVANCED PLACEMENT (AP) CALCULUS BC
Grades 11, 12

Unit of Credit: 1 Year

Pre-requisite: Pre-Calculus

Course Overview:
The topic outline for Calculus BC includes all Calculus AB topics. Additional topics that are BC topics are found in paragraphs marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their Calculus AB counterparts. Other topics may fit best after the completion of the Calculus AB topic outline. (See AP Central for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

Functions, Graphs, and Limits Analysis of Graphs

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Standard: Limits of functions (including one-sided limits).

- I can intuitively understand the limiting process.
- I can calculate limits using algebra.
- I can estimate limits from graphs or tables of data.

Standard: Asymptotic and unbounded behavior.

- I can understand asymptotes in terms of graphical behavior.
- I can describe asymptotic behavior in terms of limits involving infinity.
- I can compare relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

Standard: Continuity as a property of functions.

- I can intuitively understand the definition of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- I can understand continuity in terms of limits.
- I can understand geometric graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

*** Parametric, polar, and vector functions.** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

Derivatives

Standard: Concept of the derivative.

- I can represent graphically, numerically, and analytically the derivative.
- I can interpret the derivative as an instantaneous rate of change.
- I can define the derivative as the limit of the difference quotient.
- I can give examples of how differentiability and continuity are related.

Standard: Derivative at a point.

- I can calculate the slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- I can write an equation of the tangent line to a curve at a point and use it to calculate a linear approximation.
- I can calculate the instantaneous rate of change as the limit of the average rate of change.
- I can estimate the approximate rate of change from graphs and tables of values.

Standard: Derivative as a function.

- I can describe distinguishing characteristics of the graphs of f and f' at corresponding x-values.
- I can describe the relationship between the increasing and decreasing behavior of f and the sign of f'.
- I can state and apply the Mean Value Theorem and its geometric interpretation.
- I can write equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Standard: Second derivatives.

- I can describe corresponding characteristics of the graphs of f, f', and f''.
- I can state the relationship between the concavity of f and the sign of f''.
- I can calculate points of inflection as places where concavity changes.

Standard: Applications of derivatives.

- I can analyze curves, including the notions of monotonicity and concavity.
- + I can analyze planar curves given in parametric form, polar form, and vector form, including velocity and acceleration.
- I can optimize, both absolute (global) and relative (local) extrema.
- I can model rates of change, including related rates problems.
- I can use implicit differentiation to find the derivative of an inverse function.
- I can interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- I can interpret geometric interpretation of differential equations via direction fields and the relationship between direction fields and solution curves for differential equations.
- + I can use numerical solutions of differential equations according to Euler’s method.
- + I can apply L’Hospital’s Rule, including its use in determining limits and convergence of improper integrals and series.

Standard: Computation of derivatives.

- I can calculate derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- I can apply derivative rules for sums, products, and quotients of functions.
- I can apply the chain rule and implicit differentiation techniques.
- **+** I can calculate derivatives of parametric, polar, and vector functions.

Integrals

Standard: Interpretations and properties of definite integrals.

- I can calculate a definite integral as a limit of Riemann sums.
- I can calculate a definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:
  $\int_a^b f'(x)dx = f(b) - f(a)$.
- I can apply basic properties of definite integrals (examples include additivity and linearity).

*** Applications of integrals.**

Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid of revolution using the disk and washer methods and the shell method, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.

Standard: Fundamental Theorem of Calculus.

- I can use of the Fundamental Theorem to evaluate definite integrals.
- I can use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Standard: Techniques of antidifferentiation.

- I can evaluate antiderivatives following directly from the derivatives of basic functions.
- I can evaluate antiderivatives by substitution of variables (including changing the limits for definite integrals).
- +I can use the techniques of integration by parts and simple partial fractions (non-repeating linear factors only).
- +I can evaluate improper integrals (as limits of definite integrals).

Standard: Applications of antidifferentiation.

- I can find specific antiderivatives using initial conditions, including applications to motion along a line.
- I can solve separable differential equations and use them in modeling (including the study of the equation $y' = ky$ and exponential growth).
- + I can solve logistic differential equations and use them in modeling.

Standard: Numerical approximations to definite integrals.

- I can use Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

*Polynomial Approximations and Series

Standard: * Concept of series.

A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.

Standard: * Series of constants.

- + I can motivate examples of series, including decimal expansion.
- + I can use geometric series with some applications.
- + I can use the harmonic series.
- + I can use the alternating series and apply the error bound.
- + I can use terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of a *p*-series.
- + I can use the ratio test for convergence and divergence.
- + I can use the comparison test for series to test for convergence or divergence.

Standard: * Taylor series.

- + I can use Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve).
- + I can use the Maclaurin series and the general Taylor series centered at $x = a$.
- + I can write the Maclaurin series for the functions e^x, $\sin(x)$, $\cos(x)$, and $1/(1-x)$.
- + I can use formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.
- + I can use functions defined by power series.
- + I can calculate the radius and interval of convergence of a power series.
- + I can use the Lagrange error bound for Taylor polynomials.